Design Optimization of Linear and Non-Linear Cantilevered Energy Harvesters for Broadband Vibrations

ADAM M. WICKENHEISER

Department of Mechanical & Aerospace Engineering, George Washington University, Washington, DC, USA

ABSTRACT: In much of the vibration-based energy harvesting literature, resonant energy harvesters are designed around a single base excitation frequency, whereas many applications comprise broadband, time-varying vibrations. Since many naturally occurring vibrations are low frequency, a relatively large mass or beam length is required to resonate at the driving frequencies. This article presents a modeling and optimization procedure for designing vibration energy harvesters for maximizing power generated by vibrations recreated from real-world sources at low frequencies. It is shown that the device coupling coefficient, a significant parameter in determining the energy transduction performance, can be decoupled into terms related to the stiffness and mass distribution of the device, each of which can be optimized independently. To demonstrate the use of this design optimization procedure, measured accelerations are used to provide time-varying, broadband inputs to the energy-harvesting system. Under various size and mass constraints, optimal linear resonant harvesters are presented for human walking and automobile driving scenarios. The frequency response functions are presented alongside time histories of the power harvested using the experimental base acceleration signals. Finally, these results are compared to a non-linear device that utilizes spatially periodic magnetic excitation, a feature that is particularly suited to low-frequency, time-varying excitation.

Key Words: energy harvesting, piezoelectric, design optimization, broadband.

INTRODUCTION

THE demand for miniature wireless devices has increased dramatically in the last decade. Although on-board batteries provide a straightforward energy source for wireless devices, their relatively low energy densities make them impractical for certain applications where miniaturization is paramount (Warneke et al., 2001; Reissman et al., 2007; Wickenheiser and Garcia, 2010a). Furthermore, batteries may be infeasible due to the cost of replacement, a cost that scales with the number of devices. A means of reducing or eliminating size constraints due to the batteries and the maintenance expenses due to recharging/replacement may prove critical to the expansion of wireless technologies into new environments and applications at smaller scales.

The relatively nascent field of vibration-based energy harvesting has received significant attention due to the ubiquity of untapped vibrational energy available in or around most manmade systems (Roundy et al., 2003). The energy conversion efficiency of piezoelectric-based vibration energy harvesters has received significant attention in the literature. It has been formulated in terms of the ratio of the harvested energy to the total energy removed from the system per cycle by Shu and Lien (2006a, b), which favors devices with low mechanical damping. Alternatively, Liao and Sodano (2009) propose that the efficiency is the ratio of the energy harvested to the strain energy in the system, thus isolating the reversible mechanical energy that is successfully converted into electrical energy. Both studies identify a device's non-dimensional electromechanical coupling coefficient as the parameter to maximize for increased efficiency; however, they do not report the design parameters that correspond to its maximum.

For a fixed energy harvester design, conditions on linear circuit element values for maximum power have been reported. Optimal load resistances are given for low electromechanical coupling – in which the beam's motion is constant amplitude and independent to the circuit to which it is attached – and moderate coupling – in which the motion and voltage are assumed in phase (Lefeuvre et al., 2005). These assumptions are relaxed by Shu and Lien (2006a, b), who find the operating point for optimal power in terms of the device's natural frequency, mechanical damping, electromechanical coupling coefficient, and the load resistance. Similar results have been obtained for *RL* (Renno et al., 2009) and *RC* (Liao and Sodano, 2009) circuits. Optimal power harvesting conditions have also been derived for active circuits such as Series and Parallel Synchronized Switch Harvesting on Inductor (SSHI) (Shu et al., 2007; Lien et al., 2010;

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*Author to whom correspondence should be addressed.
E-mail: amwick@gwu.edu
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